ABSENCE OF RADIATION REACTION FOR AN EXTENDED PARTICLE IN CLASSICAL ELECTRODYNAMICS

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There are known problems with the standard Lorentz-Dirac description of radiation reaction in classical electrodynamics. The model of extended in one dimension particle is proposed and is shown that for this model there is no total change in particle momentum due to radiation reaction.

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There are known problems with the standard Lorentz-Dirac description of radiation reaction in classical electrodynamics (mass renormalization and its nonuniqueness, preacceleration, runaway solutions and so on, see, for ex., [1-7]).

In the literature one can find the opinion that some of that problems are connected with the "point" description of moving charged particle and the hope that the macroscopic model of extended particle can get rid of them (see, for ex., [8-10]).

In this article for a extended in one dimension charged particle we deduce the equation of motion and discuss some properties of particle motion along one axis.

Let us take the hydrodynamic model of an extended particle.

Then the particle is described by the mass density $m \cdot f(t,x)$, charge density $\rho(t,x)$ and current density $j(t,x)$, obeying the continuity equations

$$m\frac{\partial f}{\partial t} + m\frac{\partial (v \cdot f)}{\partial x} = 0;$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial x} = 0 \tag{1}$$

here $v = v(t,x)$ is the hydrodynamic velocity of moving extended particle.

Let the particle move under the external force $F(t,x)$ along x-axis. Then the relativistic equation of its motion reads (we choose the units $c=1$):

$$m\int dx f(t,x)\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)u=\int dx\rho(t,x)E(t,x)+\int dx f(t,x)F(t,x) \quad (2)$$

here $v=v(t,x),\ u=u(t,x)=v/\sqrt{1-v^2},\ E(t,x)$ -is the electric field, produced by moving particle (the Lorentz force is absent in one-dimension case under consideration and internal forces give zero contribution to total force):

$$E=-\frac{\partial\phi}{\partial x}-\frac{\partial A}{\partial t} \quad (3)$$

and electromagnetic potentials ϕ and A are

$$\phi(t,x)=\int dx'dt'\frac{\rho(t',x')}{|x-x'|}(a\delta_1+b\delta_2),$$

$$A(t,x)=\int dx'dt'\frac{j(t',x')}{|x-x'|}(a\delta_1+b\delta_2), \quad (4)$$

with retarded and advanced delta-functions

$$\delta_1=\delta(t'-t+|x-x'|),\quad \delta_2=\delta(t'-t-|x-x'|)$$

and a,b - constants.

Substitution of (4) in (3) and integration by parts with the help of eq. (1) (taking zero values for integrals of exact integrands in x', i.e. $\int dx'\frac{\partial}{\partial x'}(\rho\cdots)=0$, yields

$$E(t,x)=\int\frac{dx'dt'}{|x-x'|^2}\left(\rho(t',x')\frac{x-x'}{|x-x'|}(a\delta_1+b\delta_2)+j(t',x')(a\delta_1-b\delta_2)\right) \quad (5)$$

Similar integration by parts for LHS of (2) gives the common result

$$LHS=\frac{dP}{dt},\quad P=P(t)=m\int dx f(t,x)u(t,x) \quad (6)$$

here P - the particle momentum. Thus the eq. of motion reads

$$\frac{dP}{dt}=F_{self}+F_{ext},$$

$$F_{self}=\int dx\rho(t,x)E(t,x),\quad F_{ext}=\int dxf(t,x)F(t,x) \tag{7}$$

This eq. of motion has no second derivative of particle velocity; also there is no need in mass renormalization.

If the extended particle is compact, we can use in (5,7) the standard expansion in powers of $|x-x'|$ (see, for ex.,[2,3]):

$$\delta(t'-t+\epsilon|x-x'|)=\sum_{n=0}^{\infty}\frac{\epsilon^n|x-x'|^n}{n!}\frac{\partial^n}{(\partial t')^n}\delta(t'-t)$$

with $\epsilon=\pm1$.

Thus in nonrelativistic case we get the known result:

$$F_{self}=-(a+b)\int\frac{dx'dt'}{|x-x'|}\rho(t,x')\rho(t,x)\frac{\partial v(t,x')}{\partial t}+$$

$$\frac{2}{3}(a-b)\int dx'dt'\rho(t,x')\rho(t,x)\frac{\partial^2 v(t,x')}{(\partial t)^2}$$

Now consider the problem of runaway solutions and self-interaction.

The total change in particle momentum is

$$\Delta P=P(\infty)-P(-\infty)=\int dt\frac{dP}{dt}$$

and the change in particle momentum due to its self-interaction is

$$\Delta P_{self}=\int dtF_{self} \tag{8}$$

Thus

$$\Delta P=\Delta P_{self}+\int dtF_{ext} \tag{9}$$

Substitution of (5) into (7-8) gives

$$\frac{dP_{self}}{dt}=F_{ext}=\int dt'dxdx'\frac{\rho(t,x)}{|x-x'|^2}\cdot$$

$$\left(\rho(t',x')\frac{x-x'}{|x-x'|}(a\delta_1+b\delta_2)+j(t',x')(a\delta_1-b\delta_2)\right) \tag{10}$$

$$\Delta P_{self}=\int dt\,[RHS\ \ of\ \ (10)] \tag{11}$$

The solution of (1) we can write as

$$\rho(t,x) = \frac{\partial\Phi(t,x)}{\partial x}, \quad j(t,x) = -\frac{\partial\Phi(t,x)}{\partial t} \tag{12}$$

Then integration by parts in (10,11) with the help of (12) gives the following result:

$$\Delta P_{self} = \int dtdt'dxdx'\Phi(t,x)\Phi(t',x')\frac{x-x'}{|x-x'|^4}\cdot$$
$$\left[-6(a\delta_1 + b\delta_2) + 6|x-x'|\left(a\frac{\partial\delta_1}{\partial t'} + b\frac{\partial\delta_2}{\partial t}\right) - 2|x-x'|^2\left(a\frac{\partial^2\delta_1}{(\partial t')^2} + b\frac{\partial^2\delta_2}{(\partial t)^2}\right)\right] \tag{13}$$

In (13) the integrand is antisymmetric under transformations

$$t \to t', \quad t' \to t, \quad x \to x', \quad x' \to x$$

if

$$a = b \;\; (= 1/2)$$

. Then the whole integral (13) has identically zero values:

$$\Delta P_{self} = 0$$

So for an extended in one dimension particle the total change in particle momentum due to its self-interaction is zero (if is taken the half-sum of retarded and advanced interactions). Then the natural question arises - where is the source of radiated energy? - The answer is obvious and comes from eq. (9) - the source of radiated electromagnetic energy is the external force work.

Similar result holds for extended in three dimension particle.

This conclusion we formulate as the theorem:

For extended charged objects (particles) internal relativistic electromagnetic forces, also as internal forces of another nature, give zero contribution to the total change of momentum of these objects (particles).

If to go further and formulate the hypothesis:

Extended charged objects (particles) must have that form of charge density and current density, that leads to zero values of total internal relativistic electromagnetic forces, also as to zero values of total internal forces of another nature.

Then in classical electrodynamics there would be no need to use in eq. of particle motion radiation reaction force in Lorentz-Dirac or another form.

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